

Chiral-Odd Twist-3 Distribution Function $e(x)$

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Abstract

We clarify the nonperturbative origin of the δ -function singularity at $x = 0$ in the chiral-odd twist-3 distribution function $e(x)$ of the nucleon. We also compare a theoretical prediction for $e(x)$ based on the chiral quark soliton model with the empirical information extracted from the CLAS semi-inclusive DIS measurement.

1 Introduction

The subject of my talk here is the spin-independent chiral-odd twist-3 distribution function $e(x)$ of the nucleon. Why is it interesting? Firstly, its first moment is proportional to the famous πN sigma term. Secondly, within the framework of perturbative QCD, it was pointed out by Burkardt and Koike that this distribution function is likely to have a delta-function type singularity at $x = 0$ [1]. Unfortunately, the physical origin of this singularity was not fully clarified within the perturbative analysis.

The purpose of my present talk is two-fold. First, I want to show that the physical origin of this peculiar singularity is inseparably connected with the *nontrivial vacuum structure of QCD*. Secondly, I will show some theoretical predictions for this interesting quantity based on the Chiral Quark Soliton Model (CQSM), which can, for example, be compared with the recent CLAS measurement of semi-inclusive DIS scatterings [2].

2 Origin of delta-function singularity in $e(x)$

We start with the general definition of $e(x)$ given as

$$e(x) = M_N \int_{-\infty}^{\infty} \frac{dz^0}{2\pi} e^{-ixM_N z^0} E(z^0), \quad (1)$$

with

$$E(z^0) = \langle N | \bar{\psi}(-\frac{z}{2}) \psi(\frac{z}{2}) | N \rangle |_{z_3=-z_0, z_\perp=0}. \quad (2)$$

Here, the quantity $E(z_0)$ as a function of z_0 measures light-cone quark-quark correlation of scalar type in the nucleon. The existence of delta-function singularity in $e(x)$ indicates that, when z_0 becomes large, $E(z_0)$ does not damp and approaches a certain constant such that

$$E(z^0) = E^{reg}(z_0) + \text{constant}, \quad (3)$$

with $E^{reg}(z^0) \rightarrow 0$ as $z^0 \rightarrow \infty$. In fact, within the framework of the CQSM, we have analytically confirmed this behavior [3]. As proved there, the existence of this infinite-range correlation is inseparably connected with the nontrivial vacuum structure of QCD, or more concretely, the appearance of nonvanishing quark condensate.

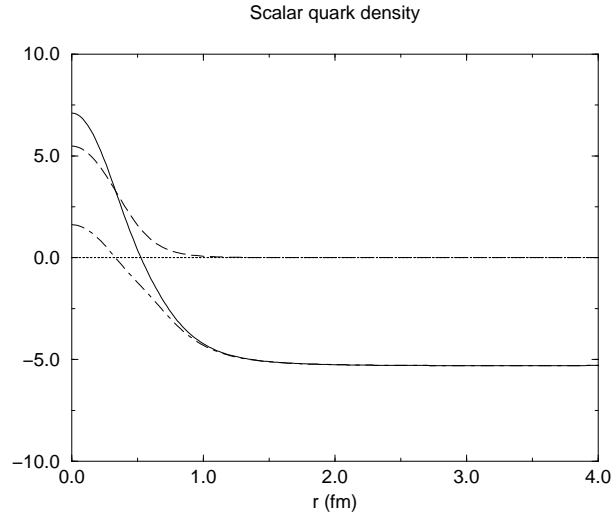


Figure 1: The scalar quark density predicted by the CQSM.

A natural question is “Why the vacuum property comes into a localized hadron observable?”. This is due to an extraordinary nature of the scalar quark density inside the nucleon illustrated in Fig.1. (This is the prediction of the CQSM.) The long-dashed curve represents the contribution of 3 valence quarks in the hedgehog mean field, while the dash-dotted curve stands for the contribution of deformed Dirac-sea quarks. One clearly sees that, as the distance from the nucleon center becomes large, the valence quark contribution damps rapidly, while the Dirac-sea contribution does not damp, and approaches some *negative constant*, which is nothing but the *vacuum quark condensate*. The existence of unending stretch of the region with nonzero scalar quark density is inseparably connected with the peculiar behavior of $E(z^0)$ as given by (3).

3 Numerical study of $e(x)$ in CQSM

Now, we turn to the numerical study of $e(x)$ within the CQSM. In this theory, the isoscalar and isovector combinations of $e(x)$ have very different N_c dependence. The former is an order N_c quantity, while the latter a subleading quantity in N_c . We need very sophisticated numerical method to handle the isoscalar distribution function $e^{(T=0)}(x)$ containing a delta-function type singularity. The detail can be found in [4]. After all, we find that the total $e^{(T=0)}(x)$ is given as a sum of the two terms, i.e. the valence quark contribution, which has a peak around $x = 1/3$, and the Dirac-sea contribution as

$$e^{(T=0)}(x) = e_{valence}^{(T=0)}(x) + e_{sea}^{(T=0)}(x). \quad (4)$$

The latter is further decomposed into two terms :

$$e_{sea}^{(T=0)}(x) = e_{sing}^{(T=0)}(x) + e_{regl}^{(T=0)}(x). \quad (5)$$

Here, the first is a singular term proportional to a delta function,

$$e_{sing}^{(T=0)}(x) = C \delta(x), \quad C \simeq 9.9, \quad (6)$$

while the second is a regular term, the magnitude of which turns out to be relatively small.

Particularly interesting here is the 1st moment sum rule for the isoscalar $e(x)$, which gives the nucleon scalar charge $\bar{\sigma}$, i.e. $\int_{-1}^1 e^{(T=0)}(x) dx = \bar{\sigma}$. Numerically, we find that

$$\bar{\sigma} = \bar{\sigma}_{valence} + \bar{\sigma}_{sea}^{regl} + \bar{\sigma}_{sea}^{sing} \simeq 1.7 + 0.18 + 9.92 \simeq 11.8, \quad (7)$$

which means that the singular term gives dominant contribution to this sum rule. With the standard values of the current quark mass $m_q \simeq (4 \sim 7) \text{ MeV}$, this nucleon scalar charge gives fairly large πN sigma term

$$\Sigma_{\pi N} \equiv m_q \bar{\sigma} \simeq (47 \sim 83) \text{ MeV}, \quad (8)$$

which favors the recent analyses of low-energy πN scattering amplitudes [5].

For the isovector part of $e(x)$, we just comment that no singularity at $x = 0$ is observed. This is reasonable since there is no isovector quark condensate in the QCD vacuum. Combining the isoscalar and isovector distributions, we can predict any of the u, d, \bar{u}, \bar{d} distributions. Fig.2 shows the comparison of the predicted flavor combination $e^u(x) + (1/4)e^{\bar{d}}(x)$ with the empirical information extracted from the CLAS data by Efremov et al. under the assumption of Collins mechanism dominance [6]. One sees that the agreement between the theory and experiment is encouraging, although it would be premature to extract any decisive conclusion only from this crude comparison on the possible violation of the pion-nucleon sigma term sum rule due to the existence of the delta-function singularity at $x = 0$.

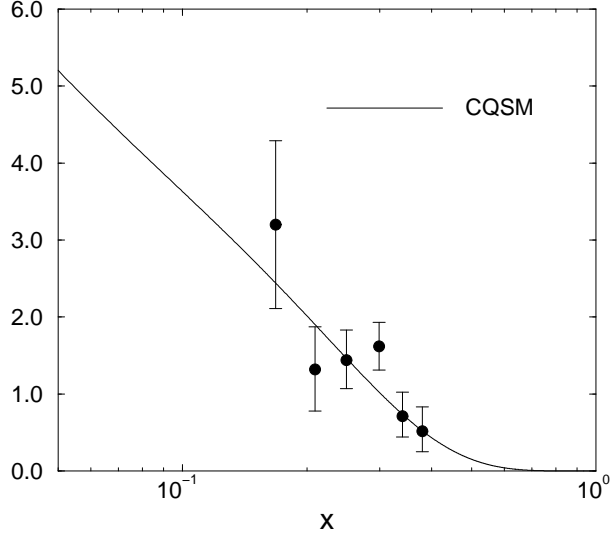


Figure 2: Comparison with a parameterization of the CLAS data.

4 Summary and Conclusion

Summarizing my talk, I have shown that the delta-function singularity in the chiral-odd twist-3 distribution $e(x)$ is a manifestation of the nontrivial vacuum structure of QCD in a hadron observable. Experimentally, the existence of this singularity will be observed as the violation of πN sigma-term sum rule of isoscalar $e(x)$. To confirm this interesting possibility, we certainly need more precise experimental information for $e(x)$ especially in the smaller x region.

References

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